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A Perturbation Theory for Light Diffraction from a Bi-grating with Multiple Surface-Polariton Excitation

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A Perturbation Theory for Light Diffraction from a Bigrating with Multiple Surface-Polariton Excitation

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Abstract

A perturbation theory has recently been developed for treating the diffration of light, with surface-polariton resonant excitation, from a bigrating surface. The theory is an approximation, to first order in the grating height, of an earlier theory based on the Rayleigh hypothesis. The perturbation theory holds for arbitrary polarization and for arbitrary plane of incidence with respect to the grating directions. It is, however, limited to treating only the possibility of two simultaneous resonant evanescent waves, whereas four are possible at normal incidence on a square grating, and three are possible on a rectangular grating. The present work generalizes the earlier work to allow for a four-fold (or three-fold) resonance. This extension also allows one now to determine the complex dispersion relation for surface-polaritons, for wavevectors at the intersection of two Brillouin zone boundary lines (not possible in the previous formulation).

I. Introduction

The study of electromagnetic wave scattering from corrugated metal surfaces, with resonant excitation of surface-plasmon-polaritons, has many important technological applications. One of these is the creation of electromagnetic radiation from an electron beam passing above a grating surface -- the Smith-Purcell effect(1). In many applications, the enormous enhancements in the absorptance at the surface and in the field intensity in the surface region, due to the grating coupling of an incident wave into a surface-polariton (or surface-electromagnetic-wave, SEW), can be further enhanced by use of a bigrating (a surface periodic in two different directions) in place of a classical grating (periodic in one direction). It was pointed out in some of the earlier theoretical studies on bigratings(2-3) that the resonant absorption of unpolarized light (e.g., of sun-light in solar absorbers) will be greater on a bigrating than on a classical grating. When the plane of incidence is normal to the groove direction, only the p-polarized component of the incident wave can excite the surface-polariton (which is itself a p-polarized surface-wave); the s-component does not couple to the surface-wave at all. By contrast, the bigrating can couple both the p- and the s- components of the incident wave, regardless of the orientation of the plane of incidence. Moreover, as demonstrated experimentally by Inagaki et al.(4), the bigrating can couple the incident wave simultaneously into two surface-polaritons, leading to an extra absorptance, for incidence orientations in which the classical grating would only excite a single polariton. These same processes indicate mechanisms by which the Smith-Purcell radiation can also be enhanced by using a bigrating in place of a classical grating.

The first theory of the Smith-Purcell effect that was both mathematically exact and physically realistic was the work of P.M. van den Berg(5). This theory used a Green's function approach, proposed by di Francia(6), in which the component evanescent waves, which make up the Fourier expansion of the field of the electron current, are seen as exciting the various diffracted orders of the classical grating.

One shortcoming of the van den Berg theory is that it considers the metal on which the grating is ruled to be a perfect conductor. This approximation precludes the possibility of accounting for the effects of the surface-polaritons and their resonant excitation. Recently, Chuang and Kong(7) developed a theory which considers a sinusoidal grating on silver, described by a complex dielectric function, and uses an extended boundary condition method to calculate the radiated energy.

Similar techniques are *in principle* applicable to the treatment of Smith-Purcell radiation from a bigrating, but the numerical difficulties can become prohibitive: the computing time necessary to calculate the diffracted field, arising from each Fourier component in the incident field, will be much greater for the two-dimensional case, the bigrating, than for the one-dimensional case, the classical grating. Thus perturbative techniques are desired.

A non-perturbative theory of light diffraction from a bigrating, with surface-polariton resonances, was developed by Glass, Maradudin, and Celli(8). This theory uses the vectorial form of Kirchhoff's integral to eliminate the field in the metal and thereby to cut in half the size of the matrix equation to be solved. There are nevertheless very large matrix equations to be solved with this method: from 98x98 up to 243x243 in Ref. 8. Thus, a perturbation theory was recently developed by Glass(9), which applies the coupled-mode theory of Glass, Weber, and Mills(10) to the exact theory of Glass, Maradudin, and Celli(8). This perturbation theory reduces the computations to a 4x4 matrix equation, by retaining, to first-order, the two amplitudes (p-and s- polarized components) of the specularly diffracted beam and two evanescent-wave amplitudes (p-components only) corresponding to the possible simultaneous resonant excitation of two surface-polaritons. These two surface-polaritons may be propagating in non-collinear directions (unlike the case for the classical grating, in which two polaritons can be excited only for normal incidence, and then are propagating in exactly opposing directions). This theory as implemented in Ref.9 has one shortcoming: it cannot treat the case of more than two surface-polaritons excited simultaneously. For example, it cannot treat the cases of normal incidence on a square-grating when there are four surface-polaritons excited simultaneously or of oblique incidence on a rectangular grating when there are three simultaneous excitations. This shortcoming could be an important one if one wishes to use these perturbative approaches in calculating the Smith-Purcell radiation. Fortunately, it is not a difficult problem to correct. A straight-forward extension of the theory of Ref. 9 will produce a 6x6 matrix equation for four resonant-wave (p-) amplitudes and the two specular beam amplitudes. It is the purpose of this report to present the results of the extension of the perturbation theory of Ref. 9 to treat the cases in which there are up to four surface excitations.

II. Theory

The physical geometry is as follows. The nominal flat surface is in the plane $x_3=0$. The surface profile is described by $x_3=\zeta(\vec{x}_\parallel)$ [where $\vec{x}_\parallel=\hat{x}_1 x_1+\hat{x}_2 x_2$]. The region $x_3>\zeta(\vec{x}_\parallel)$ is vacuum; $x_3<\zeta(\vec{x}_\parallel)$ is dielectric, described by the complex dielectric function $\epsilon(\omega)=\epsilon_R(\omega)+i\epsilon_I(\omega)$. The surface profile function is periodic in two directons. Light of frequency ω and wavevector $\vec{k}$ is incident from the vacuum (with angle of incidence θ). The projection of $\vec{k}$ onto the surface is $\vec{k}_\parallel$, which makes an angle ϕ with $\hat{x}_1$.

The electric field in the vacuum is

$$\vec{E}(\vec{x}) = \vec{E}^i(\omega,\vec{k}_\parallel)\,\exp[i\vec{k}_\parallel\cdot\vec{x}_\parallel - i\alpha_o(\omega,k_\parallel)x_3] + \\ + \sum_{\vec{G}} \vec{E}^s(\omega,\vec{K}_{\vec{G}})\,\exp[i\vec{K}_{\vec{G}}\cdot\vec{x}_\parallel + i\alpha_o(\omega,K_{\vec{G}})x_3] \quad . \qquad (1)$$

In the above equation we have:

$$\vec{K}_{\vec{G}} = \vec{k}_{\|} + \vec{G} \quad , \tag{2}$$

where $\vec{G}$ is a reciprocal lattice vector defined by the bigrating periodicity, which for a simple square-lattice grating is

$$\vec{G} = \frac{2\pi}{a}\,(m_1\hat{x}_1 + m_2\hat{x}_2) \qquad \text{for} \quad m_j=0,\pm 1,\pm 2,\ldots \tag{3}$$

("a" is the period); and where

$$\vec{k}_{\|} = \vec{K}_0 = \frac{\omega}{c}\,(\cos\phi\ \hat{x}_1 + \sin\phi\ \hat{x}_2)\sin\theta \quad . \tag{4}$$

Also appearing in Eq.(1) is

$$\alpha_0(\omega,K_{\vec{G}}) = \begin{cases} (\omega^2/c^2 - K_{\vec{G}}^2)^{1/2}, & \text{for} \quad K_{\vec{G}}^2 < \omega^2/c^2 \qquad (5a) \\ i(K_{\vec{G}}^2 - \omega^2/c^2)^{1/2}\ , & \text{for} \quad K_{\vec{G}}^2 > \omega^2/c^2 \qquad (5b) \end{cases} \quad .$$

The amplitudes of the incident and scattered fields are:

$$\vec{E}^i(\omega,\vec{k}_{\|}) = [\hat{k}_{\|}+\hat{x}_3\,\frac{k_{\|}}{\alpha_o(\omega.k_{\|})}]B_{\|} + (\hat{x}_3\ \mathbf{X}\ \hat{k}_{\|})B_{\perp} \tag{6}$$

and

$$\vec{E}^s(\omega,\vec{K}_{\vec{G}}) = [\hat{K}_{\vec{G}} - \hat{x}_3\,\frac{K_{\vec{G}}}{\alpha_o(\omega,K_{\vec{G}})}\,]A_{\|}(\omega,\vec{K}_{\vec{G}}) + (\hat{x}_3\ \mathbf{X}\ \hat{K}_{\vec{G}})A_{\perp}(\omega,\vec{K}_{\vec{G}}) \tag{7}$$

The p- and s- components for the incident wave are expressed in terms of $B_{\|}$ and $B_{\perp}$, and for the scattered waves in terms of the Rayleigh coefficients $A_{\|}$ and $A_{\perp}$.

The Rayleigh coefficients are the unknowns in the problem. To find them one could expand the field in the metal in a Rayleigh expansion and then apply the boundary conditions at the grating surface (this is the Rayleigh method). The method used in Ref. 8 reduces the number of equations in half by employing the vectorial equivalent of the Kirchhoff integral to eliminate the field in the metal. The effective boundary condition that results then yields two infinite sets of linear algebraic equations for the unknown coefficients. These are Eqs.(2.18a) and (2.18b) in Ref. 8 and equations (8a) and (8b) in Ref 9.

The equations for the Rayleigh coefficients contain the integral:

$$I(\alpha|\vec{G}) = \frac{1}{a_c} \iint_{\text{cell}} d^2x_{\parallel} \exp[-i\vec{G}\cdot\vec{x}_{\parallel} - i\alpha\zeta(\vec{x}_{\parallel})] \quad , \tag{8}$$

where a_c is the area of a unit cell on the bigrating surface, over which the integral is performed.

The perturbation theory begins by expanding the integrand of "I" to first order in ζ, thus yielding:

$$I(\alpha|\vec{G}) = \delta_{\vec{G},0} - i\alpha\tilde{\zeta}(\vec{G}) \tag{9}$$

where

$$\tilde{\zeta}(\vec{G}) = \frac{1}{a_c} \iint_{\text{cell}} d^2x_{\parallel}\zeta(\vec{x}_{\parallel})\exp(-i\vec{G}\cdot\vec{x}_{\parallel}) \quad . \tag{10}$$

Under the assumption that the dimensionless parameter $\alpha\zeta$ is small, we can substitute equation (9) into the equations for the A's, and (assuming that $\tilde{\zeta}(0)=0$) find the first-order results:

$$A_{\parallel}(\vec{G}) = i\,\frac{\alpha_{\vec{G}\vec{G}}}{c_{\vec{G}\vec{G}}} \sum_{\vec{G}'}{}' \tilde{\zeta}(\vec{G}-\vec{G}')\,[c_{\vec{G}\vec{G}'}A_{\parallel}(\vec{G}') - a_{\vec{G}\vec{G}'}A_{\perp}(\vec{G}')]$$

$$- \frac{\alpha_{\vec{G}\vec{G}}}{c_{\vec{G}\vec{G}}} \left[\frac{\delta_{\vec{G},0} - i\beta_{\vec{G}}\,\tilde{\zeta}(\vec{G})}{\beta_{\vec{G}}} \right] [e_{\vec{G}}\,B_{\parallel} - a_{\vec{G},0}B_{\perp}] \tag{11a}$$

and

$$A_{\perp}(\vec{G}) = i\alpha_{\vec{G}\vec{G}} \sum_{\vec{G}'}{}' \tilde{\zeta}(\vec{G}-\vec{G}')\,[a_{\vec{G}\vec{G}'}A_{\parallel}(\vec{G}') + b_{\vec{G}\vec{G}'}A_{\perp}(\vec{G}')]$$

$$- \alpha_{\vec{G}\vec{G}} \left[\frac{\delta_{\vec{G},0} - i\beta_{\vec{G}}\,\tilde{\zeta}(\vec{G})}{\beta_{\vec{G}}} \right] [a_{\vec{G},0}\,B_{\parallel} + b_{\vec{G},0}B_{\perp}] \quad . \tag{11b}$$

Here the prime on the summation indicates that $\vec{G}' \neq \vec{G}$, and we have employed the notation $A_{\parallel}(\vec{G})$ for $A_{\parallel}(\omega,\vec{K}_{\vec{G}})$ (similarly for $A_{\perp}$) and have used the following definitions:

$$\beta_{\vec{G}} = \alpha(\omega, K_{\vec{G}}) + \alpha_o(\omega, k_{\parallel}) \tag{12a}$$

$$\alpha_{\vec{G}\vec{G}'} = \alpha(\omega, K_{\vec{G}}) - \alpha_o(\omega, K_{\vec{G}'}) \tag{12b}$$

$$a_{\vec{G}\vec{G}'} = \hat{x}_3 \cdot (\hat{K}_{\vec{G}} \mathbf{X} \hat{K}_{\vec{G}'}) \tag{12c}$$

$$b_{\vec{G}\vec{G}'} = \hat{K}_{\vec{G}} \cdot \hat{K}_{\vec{G}'} \tag{12d}$$

$$c_{\vec{G}\vec{G}'} = b_{\vec{G}\vec{G}'} + \frac{K_{\vec{G}} K_{\vec{G}'}}{\alpha(\omega, K_{\vec{G}})\alpha_o(\omega, K_{\vec{G}'})} \tag{12e}$$

$$e_{\vec{G}} = \hat{K}_{\vec{G}} \cdot \hat{k}_{\parallel} - \frac{K_{\vec{G}} k_{\parallel}}{\alpha(\omega, K_{\vec{G}})\alpha_o(\omega, k_{\parallel})} \tag{12f}$$

$$\alpha(\omega, K_{\vec{G}}) = \left[\epsilon(\omega)\frac{\omega^2}{c^2} - K_{\vec{G}}^2\right]^{1/2} \tag{12g}$$

As in Ref.9, we assume that the amplitudes of the two components of the specular beam, $A_{\parallel}(0)$ and $A_{\perp}(0)$, can dominate the diffracted orders. Unlike the case considered in Ref.9, we now consider the possibility of having four, not two, resonant evanescent waves. We may have four wavevectors in the surface plane, corresponding to four different reciprocal lattice vectors, which satisfy the resonance condition:

$$\vec{K}_{\vec{G}_r} = \vec{k}_{\parallel} + \vec{G}_r = \vec{K}_{sp}(\omega) \quad , \qquad \text{for } r=1,\ldots,4 \tag{13}$$

where $\vec{K}_{sp}(\omega)$ is a surface-polariton wavevector corresponding to the frequency of the incident light. To zeroth-order in the roughness (the flat surface limit), the locus of points $\vec{K}_{sp}(\omega)$, for a given frequency, defines a circle. Hence, to zeroth-order the four vectors $\vec{K}_{\vec{G}}$ lie on a circle, at the four-fold resonance. If any one, or all four, of the $\vec{K}_{\vec{G}r}$ are at resonance, then for that, or those, wavevectors we have to zeroth-order:

$$\alpha(\omega, K_{\vec{G}_r}) + \epsilon(\omega)\alpha_o(\omega, K_{\vec{G}_r}) = 0 \tag{14}$$

We are therefore going to assume that six amplitudes can be larger than all the rest, these being $A_{\parallel}(0)$, $A_{\perp}(0)$, $A_{\parallel}(\vec{G}_1)$, $A_{\parallel}(\vec{G}_2)$, $A_{\parallel}(\vec{G}_3)$, and $A_{\parallel}(\vec{G}_4)$; and for each of these we can write an explicit equation, using Eq.(11), with the six important terms separated out from the sum on the right hand side. For $A_{\parallel}(1)$ this procedure gives us:

$$\begin{aligned} A_{\parallel}(1) = \frac{\alpha_{11}}{c_{11}}\, i \Bigg[& \sum_{p=2,3,4} \tilde{\zeta}(1-p) c_{1p} A_{\parallel}(p) + \tilde{\zeta}(1)[c_{10}A_{\parallel}(0) - a_{10}A_{\perp}(0)] \\ & + \tilde{\zeta}(1)[e_1 B_{\parallel} - a_{10}B_{\perp}] + \Big\{ \sum_{j}{}' \tilde{\zeta}(1-j)[c_{1j}A_{\parallel}(j) - a_{1j}A_{\perp}(j)] \\ & \qquad - \sum_{p=2,3,4} \tilde{\zeta}(1-p)\, a_{1p}A_{\perp}(p) \Big\} \Bigg] \end{aligned} \tag{15}$$

To simplify the notation we replace the variable or subscript $\vec{G}_j$ everywhere by j alone (where j=1,..,4 are the resonant beams and j=0 the specular beam). The prime on the summation indicates to sum on all $j \neq 0,1,2,3,4$. The nonresonant terms have now been segregated in the brackets on the right hand side. The equation for $A_{\parallel}(2)$ is obtained from Eq.(15) by simply interchanging the 1 and 2; and similarly for $A_{\parallel}(3)$ and $A_{\parallel}(4)$.

The specular amplitudes are:

$$\begin{aligned} A_{\perp}(0) = {} & i\alpha_{00} \sum_{r=1}^{4} \tilde{\zeta}(-r) a_{0r} A_{\parallel}(r) - \frac{\alpha_{00}}{\beta_0} B_{\perp} \\ & + i\alpha_{00} \Big\{ \sum_{j}{}' \tilde{\zeta}(-j)[a_{0j}A_{\parallel}(j) + b_{0j}A_{\perp}(j)] + \sum_{r=1}^{4} \tilde{\zeta}(-r) b_{0r} A_{\perp}(r) \Big\} \end{aligned} \tag{16}$$

$$A_\parallel(0) = i\frac{\alpha_{00}}{c_{00}} \sum_{r=1}^{4} \tilde{\zeta}(-r)\, c_{0r} A_\parallel(r) - \frac{e_0 \alpha_{00}}{\beta_0 c_{00}} B_\parallel$$

$$+ i\frac{\alpha_{00}}{c_{00}} \left\{ \sum_{j}{}' \tilde{\zeta}(-j)\,[c_{0j} A_\parallel(j) - a_{0j} A_\perp(j)] - \sum_{r=1}^{4} \tilde{\zeta}(-r)\, a_{0r} A_\perp(r) \right\} . \qquad (17)$$

The amplitude of a given nonresonant wave can also be expressed with Eq.(11), but now we drop all the other nonresonant terms from the sum on the right hand side:

$$A_\perp(j) = i\alpha_{jj} \left\{ \sum_{p=1}^{4} \tilde{\zeta}(j-p)\, a_{jp} A_\parallel(p) + \tilde{\zeta}(j)\,[a_{j0} A_\parallel(0) + b_{j0} A_\perp(0)] \right.$$

$$\left. + \tilde{\zeta}(j)\,[a_{j0} B_\parallel + b_{j0} B_\perp] \right\} \qquad (18)$$

$$A_\parallel(j) = \frac{i\alpha_{jj}}{c_{jj}} \left\{ \sum_{p=1}^{4} \tilde{\zeta}(j-p)\, c_{jp} A_\parallel(p) + \tilde{\zeta}(j)\,[c_{j0} A_\parallel(0) - a_{j0} A_\perp(0)] \right.$$

$$\left. + \tilde{\zeta}(j)\,[e_j B_\parallel - a_{j0} B_\perp] \right\} . \qquad (19)$$

We now follow the development given in Ref.(9). We substitute Eqs.(18) and (19) for the nonresonant amplitudes back into Eq.(15) for the the resonant amplitude A(1); and for the denominator c_{11} we use

$$c_{11} = 1 + \frac{K_1^2}{\alpha(1)\alpha_o(1)} = \frac{\alpha(1)\alpha_o(1) + K_1^2}{\alpha(1)\alpha_o(1)} , \qquad (20)$$

where $\vec{K}_j$ now stands for $\vec{K}_{\vec{G}j}$ and $\alpha(j)$ for $\alpha(\omega, K_{\vec{G}j})$. The denominator c_{11} vanishes to zeroth-order at the $\vec{K}_1$-resonance. We multiply both sides of the resulting equation for $A_\parallel(1)$ by $\alpha^2(1)\alpha_o^2(1) - K_1^4$, in the form

$$\alpha^2(1)\alpha_o^2(1) - K_1^4 = [\alpha(1)\alpha_o(1) - K_1^2]\;[\alpha(1)\alpha_o(1) + K_1^2] \qquad (21)$$

on the right hand side and as

$$\alpha^2(1)\alpha_o^2(1)-K_1^4 = \epsilon\,\frac{\omega^2}{c^2}\left(\frac{\omega^2-\omega_1^2}{c^2}\right) \quad . \tag{22}$$

on the left hand side. Here we have

$$\omega_r^2 = \left(\frac{1+\epsilon}{\epsilon}\right)c^2K_r^2 \quad , \tag{23}$$

which is the frequency of a surface-polariton at wavevector $\vec{K}_r$ on the flat surface. Finally, we note that in the equation which we now have for $A_{\parallel}(1)$, we are left with an $A_{\parallel}(1)$ on the right hand side in a term proportional to $\tilde{\zeta}^2$, which arose from the nonresonant terms. We bring this term over to the left side, and use it to renormalize the flat-surface polariton frequency:

$$\tilde{\omega}_r^{\;2} = \omega_r^{\;2}\left\{1-\frac{c^2\alpha_{rr}\gamma_r}{(1+\epsilon)\omega^2K_r^{\;2}}\left[\sum_{\substack{p=1\\(p\neq r)}}^{4}\tilde{\zeta}(r-p)\tilde{\zeta}(p-r)\alpha_{pp}a_{rp}^2 + \sum_j{}' \,\tilde{\zeta}(r-j)\tilde{\zeta}(j-r)\alpha_{jj}\left[\frac{c_{rj}c_{jr}}{c_{jj}}+a_{rj}^2\right]\right]\right\}$$

for r=1,2,3, or 4. (24)

Here we employ the definition:

$$\gamma_r = \alpha(r)\alpha_o(r)[\alpha(r)\alpha_o(r) - K_r^{\;2}] \tag{25}$$

Next we go back to Eqs.(18) and (19) for the nonresonant terms and substitute them into Eqs.(16) and (17) for $A_{\perp}(0)$ and $A_{\parallel}(0)$.

At this point we have six linear algebraic equations for $A_{\parallel}(0)$, $A_{\perp}(0)$, and $A_{\parallel}(r)$ [r=1,...,4] in terms of $B_{\parallel}$ and $B_{\perp}$; expressed in matrix form these equations are:

$$\begin{bmatrix} (\Omega^2-\tilde{\Omega}_1{}^2) & L_{12} & L_{13} & L_{14} & M_1 & N_1 \\ L_{21} & (\Omega^2-\tilde{\Omega}_2{}^2) & L_{23} & L_{24} & M_2 & N_2 \\ L_{31} & L_{32} & (\Omega^2-\tilde{\Omega}_3{}^2) & L_{34} & M_3 & N_3 \\ L_{41} & L_{42} & L_{43} & (\Omega^2-\tilde{\Omega}_4{}^2) & M_4 & N_4 \\ O_1 & O_2 & O_3 & O_4 & (1+P) & Q \\ R_1 & R_2 & R_3 & R_4 & S & (1+T) \end{bmatrix} \cdot \begin{bmatrix} A_{\parallel}(1) \\ A_{\parallel}(2) \\ A_{\parallel}(3) \\ A_{\parallel}(4) \\ A_{\parallel}(0) \\ A_{\perp}(0) \end{bmatrix}$$

$$= \begin{bmatrix} (U_1-M_1)B_{\parallel} - N_1B_{\perp} \\ (U_2-M_2)B_{\parallel} - N_2B_{\perp} \\ (U_3-M_3)B_{\parallel} - N_3B_{\perp} \\ (U_4-M_4)B_{\parallel} - N_4B_{\perp} \\ (W+V-P)B_{\parallel} - QB_{\perp} \\ (X-S)B_{\parallel} - (T+\frac{\alpha_{00}}{\beta_0})B_{\perp} \end{bmatrix} . \qquad (26)$$

In this equation $\Omega = \omega/(2\pi c/a) = a/\lambda$; $\tilde{\Omega}_r = \tilde{\omega}_r/(2\pi c/a)$; while the other symbols, namely the capital letters L through X, are defined in the appendix.

The problem now is to solve, numerically, this set of six equations for the six unknown coefficients, the A's. Then these six coefficients can be substituted back into Eqs.(18) and (19) to find the nonresonant coefficients. With all the coefficients thus at hand, we can then determine the scattered field vector-amplitude, $\vec{E}^S(\vec{K}_{\vec{G}})$, from Eq.(7) -- for each diffracted order and for each evanescent wave. We can thus obtain the reflectance, or efficiency, associated with each of the grating's diffracted orders, as well as the total field and field-enhancement near the grating surface.

III. Conclusion

The analytic formulation of a perturbation theory has been presented here, in complete analogy to that developed in Ref.9 , in order to treat the special case, of normal incidence on the bigrating surface, which was not treated in Ref.9 . Numerical implementation of the theory, in direct analogy to that in Ref.9 is now being carried out. This implementation will include a comparison of the perturbation theory to the exact ,i.e., non-perturbative, theory, in order to test the limits of validity of the former. When the conditions at normal incidence are such as to excite simultaneously four surface-polaritons, the absorptance is expected to be enhanced approximately four-fold over that found when a single surface-polariton is resonantly excited.

Again, in analogy to the discussion in Ref.9, the results presented here can be used to calculate the dispersion relation for surface-polaritons on bigratings. We take the matrix on the left hand side of Eq.(26), cross out the fifth and sixth rows and columns (corresponding to the specular beams), take the determinant of the resulting 4x4 matrix, and set it equal to zero. The primed summation symbols that appear in the definitions of the matrix elements must first be redefined to reinclude the specular beams, j=0. Solution of the determinantal equation by fixing $\vec{k}_{||}$

and solving for complex ω will then yield both the dispersion and the life-time of the surface-polariton. The present formulation, unlike that in Ref.9, will enable us to treat those points in the $\vec{k}_{\parallel}$-plane where two Brillouin zone boundary lines intersect, that is, where there are *four* coupled surface polaritons. Work on numerically solving the dispersion relation is just getting started.

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Appendix

Eq.(26) employs the following definitions for the matrix elements.

$$L_{rp} = h_r\left[i\tilde{\zeta}(r-p)c_{rp} - \sum_j{}' \tilde{\zeta}(r-j)\tilde{\zeta}(j-p)(f_{rj}c_{jp}-a_{rj}a_{jp})\alpha_{jj}\right] \qquad \text{(A-1)}$$

$$M_r = h_r\Bigg[i\tilde{\zeta}(r)c_{r0} + \sum_{\substack{p=1 \\ (\neq r)}}^{4} \tilde{\zeta}(r-p)\tilde{\zeta}(p)\alpha_{pp}a_{rp}a_{p0} - \sum_j{}' \tilde{\zeta}(r-j)\tilde{\zeta}(j)(f_{rj}c_{j0}-a_{rj}a_{j0})\alpha_{jj}\Bigg] \qquad \text{(A-2)}$$

$$N_r = -h_r\Big[i\tilde{\zeta}(r)a_{r0} - \sum_{\substack{p=1\\(\neq r)}}^{4} \tilde{\zeta}(r-p)\tilde{\zeta}(p)\alpha_{pp}a_{rp}b_{p0} - \sum_{j}{}' \, \tilde{\zeta}(r-j)\tilde{\zeta}(j)(f_{rj}a_{j0}+a_{rj}b_{j0})\alpha_{jj}\Big] \tag{A-3}$$

$$O_r = \frac{-\alpha_{00}}{c_{00}}\Big[i\tilde{\zeta}(-r)c_{0r} + \sum_{\substack{p=1\\(\neq r)}}^{4} \tilde{\zeta}(p-r)\tilde{\zeta}(-p)\alpha_{pp}a_{pr}a_{0p} - \sum_{j}{}' \, \tilde{\zeta}(j-r)\tilde{\zeta}(-j)(f_{0j}c_{jr}-a_{rj}a_{j0})\alpha_{jj}\Big] \tag{A-4}$$

$$P = \frac{\alpha_{00}}{c_{00}}\Big[\sum_{p=1}^{4} \tilde{\zeta}(p)\tilde{\zeta}(-p)\alpha_{pp}a_{op}^{2} + \sum_{j}{}' \, \tilde{\zeta}(j)\tilde{\zeta}(-j)(f_{0j}c_{j0}+a_{0j}^{2})\alpha_{jj}\Big] \tag{A-5}$$

$$Q = -\frac{\alpha_{00}}{c_{00}}\Big[\sum_{p=1}^{4} \tilde{\zeta}(p)\tilde{\zeta}(-p)\alpha_{pp}a_{op}b_{p0} + \sum_{j}{}' \, \tilde{\zeta}(j)\tilde{\zeta}(-j)(f_{0j}-b_{j0})a_{j0}\alpha_{jj}\Big] \tag{A-6}$$

$$R_r = -\alpha_{00}\Big[i\tilde{\zeta}(-r)a_{0r} - \sum_{\substack{p=1\\(\neq r)}}^{4} \tilde{\zeta}(p-r)\tilde{\zeta}(-p)\alpha_{pp}a_{pr}b_{p0} - \sum_{j}{}' \, \tilde{\zeta}(j-r)\tilde{\zeta}(-j)(f_{jr}a_{0j}+a_{jr}b_{j0})\alpha_{jj}\Big] \tag{A-7}$$

$$S = \alpha_{00}\Big[\sum_{p=1}^{4} \tilde{\zeta}(p)\tilde{\zeta}(-p)\alpha_{pp}a_{p0}b_{p0} + \sum_{j}{}' \, \tilde{\zeta}(j)\tilde{\zeta}(-j)(f_{j0}-b_{j0})a_{0j}\alpha_{jj}\Big] \tag{A-8}$$

$$T = \alpha_{00}\Big[\sum_{p=1}^{4} \tilde{\zeta}(p)\tilde{\zeta}(-p)\alpha_{pp}b_{p0}^{2} + \sum_{j}{}' \, \tilde{\zeta}(j)\tilde{\zeta}(-j)\Big(\frac{a_{0j}^{2}}{c_{jj}} + b_{j0}^{2}\Big)\alpha_{jj}\Big] \tag{A-9}$$

$$U_r = h_r\Big[i\tilde{\zeta}(r)g_r - \sum_{j}{}' \, \tilde{\zeta}(r-j)\tilde{\zeta}(j)\tau_j c_{rj}\Big] \tag{A-10}$$

$$V = \frac{\alpha_{00}}{c_{00}}\sum_{j}{}' \, \tilde{\zeta}(j)\tilde{\zeta}(-j)\tau_j c_{0j} \tag{A-11}$$

$$W = -\frac{\alpha_{00}e_0}{c_{00}\beta_0} \tag{A-12}$$

$$X = \alpha_{00}\sum_j{}' \tilde{\zeta}(j)\tilde{\zeta}(-j)\tau_j a_{0j} \tag{A-13}$$

In the above equations the following additional definitions have been introduced:

$$f_{mj} = \frac{c_{mj}}{c_{jj}} \tag{A-14a}$$

$$f_{jm} = \frac{c_{jm}}{c_{jj}} \tag{A-14b}$$

$$g_j = c_{j0} - e_j \tag{A-15}$$

$$h_r = -\frac{\gamma_r\alpha_{rr}c^4}{\epsilon\omega^2}\left[\frac{1}{2\pi c/a}\right]^2 \tag{A-16}$$

$$\tau_j = g_j\alpha_{jj}/c_{jj} \tag{A-17}$$

The terms a_{ij}, b_{ij}, c_{ij}, e_j were defined in the text, Eq.(12). They are dimensionless, as are f_{ij} and g_j . The summations Σ' are, as in the text, over all j except j=0,1,2,3,and 4 .

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